**Towards a definition of ‘rate’**

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Question: How do we do justice in the BFO framework to rates of change, which seem intuitively to be attributes of processes in some ways analogous to BFO:qualities?

To answer this question we consider the specific case of beating processes, as for example of a heart.

How do we do justice to the fact that, intuitively, a process of heart beating can change its rate from one time to the next.

Answer: There are two sub-processes (by 5.), one spanning the first time, the other spanning the second time, and the rate of one is different from the rate of the other.

What follows is an attempt to make this work, given that there are beating processes which change continuously (so that we have to make sense of a beating process boundary having a certain rate *at an instant*)

p instanceOf beating-at-n-bpm =def.

**Case 1** (see Figure 1): p is regular, and extends across at least one cycle

there is some decomposition of p into similar process parts p = sum of p1, …, pn, for n ≥ 1, which is such that each of the pi projects onto a time interval of time 1/n

Figure 1

**Case 2** (see figure 2): the rate of beating of a process is varying, but for a certain interval t1 (from 1.5 to 2 seconds), which is less than the extent of the relevant cycle, it is beating at a rate of n bpm.

Here it is as if at the relevant time the process could be extended to Case 1. We need to say something like:

during this interval the beating process is similar to a 0.5 second long segment of a process that is otherwise similar and is beating at n bpm according to our definition. Perhaps like this:

p instance of partial\_interval\_beating-at-n bpm =def. there is some time interval t, p projects onto t, there is some process q such that q instance\_of beating-at-n bpm and p is similar to a segment of q that is of length equal to that of t

Figure 2

**Case 3** (see Figure 3) Constant acceleration (in every minute the rate rises by a certain amount): there might be an instantaneous process (process boundary) p1 to which we can assign a rate of n bpm. To this end we need to use the idea of limits; however small an interval around n we choose, we can find an interval around t1 in which the beating process is arbitrarily closely similar to a process that is an instance of partial\_interval\_beating-at-n bpm.

Thus suppose that the heart is beating as in Figure 3 and that its rate of beating is decreasing continuously between t1 and t2. We want to say that at the mid-point the heart is beating at 64 bpm. Yet by our definitions above at no time in the given interval do we have an instance of beating-at-64-bpm in either Case 1 or Case 2. Remember that we cannot *make* time-indexed instantiation assertions concerning processes at all.

We first define

two beating processes are δ-similar (meaning: have δ-similar beat rates) =def. the difference between their rates (defined under either case 2 or case 3) is less than or equal to δ bpm.

We then define

p instance of instantaneous\_beating-at-n bpm =def. p is a process boundary (instantaneous process part) in the interior of some process p1 and given any δ we can find some process p2 such that p2 interior part of p and p2 part of p1 & p1 δ-similar to some process that is an instance of partial\_interval\_beating-at-n bpm

Figure 4

Case 4: Beating at a rate increasing by a fixed amount (see Figure 4) (this applies also to Figure 5)

**p instance of beating\_at\_a\_rate\_increasing\_by n bpm2**

(i.e. in every minute the rate rises by an increasing amount)

=def. for any instantaneous process boundary p1 in the interior of p and for any m1, if p1 instance of instantaneous\_beating-at-m1 bpm & p1 projects onto t1, then for any δ there is some instantaneous process boundary p2 and some m2 such that p2 instance of instaneous\_beating-at-m2 & p2 projects onto t2 and p1 δ-similar to some process that is an instance of partial\_interval\_beating-at-n bpm & the difference between nand

is less than δ

Figure 4

**The master argument for profiles now reads as follows:**

‘Similar’ in all of the above has to mean: have a similar beating-motion profile

For suppose that each of the pi involves e.g. the making of a noise of a quite different sort, or the flashing of a color of a quite different sort, or any other kind of change.

Footnote:

The idea, illustrated by Figures 1-3, is to compute the rate of change as the [limiting value](http://en.wikipedia.org/wiki/Limit_of_a_function) of the [ratio of the differences](http://en.wikipedia.org/wiki/Difference_quotient) Δ*y* / Δ*x* as Δ*x* becomes infinitely small.

In [Leibniz's notation](http://en.wikipedia.org/wiki/Leibniz%27s_notation), such an [infinitesimal](http://en.wikipedia.org/wiki/Infinitesimal) change in *x* is denoted by *dx*, and the derivative of *y* with respect to *x* is written

 \frac{dy}{dx} \,\!